Homework 2: Due Thursday, January 31

1. **(a)** Consider a disk with an azimuthally symmetric distribution

$$\rho(R, z) = \frac{\Sigma_0}{h_Z} e^{-|z|/h_Z} e^{-R/h_R}$$

with Σ_0, h_Z, and h_R all constants. Determine the mass of the disk in terms of these paramters and rewrite the density in terms of this total mass and Σ_0, h_R.

(b) Define $M(r)$ to be the mass contained within a *spherical* volume of radius r. Compute $M(r)$ for the distribution of part (a).

(c) Assume (incorrectly) that the gravitational field due to this mass can be computed by putting a point mass $M(r)$ at the center of the galaxy. Write down an analytic expression for the circular velocity as a function of radius under this assumption.

2. In class, we treated the above problem without making the spherical approximation and found that

$$v^2 = \pi G \Sigma_0 \frac{R^2}{h_R} \left[I_0(y) K_0(y) - I_1(y) K_1(y) \right],$$

where $y \equiv R/2h_R$. Plot the velocity as a function of R/h_R using this exact expression. Compare with your result from Problem 1. What is the largest difference between the two curves?

3. Fit a thin disk plus dark matter halo profile to the rotational velocities measured for NGC2403. Take an isothermal profile we discussed in class, with $\rho_{\rm dm} = \rho_0[1 + (r/r0)^2]^{-1}$ with $r_0 = 5$kpc and ρ_0 a free parameter. The velocity induced by the dark matter can be written down analytically; for that induced by the disk you can use either the approximation of Problem 1 or the exact result if you coded it up in Problem 2.

(a) Get the data from either

`http://www.ioa.s.u-tokyo.ac.jp/~sofue/RC99/2403.dat`

or

`http://home.fnal.gov/~dodelson/242/2403.dat`

Plot v vs. R. Note from the file that $h_R = 2.13$ kpc.

(b) Define the χ^2 for this data and model:

$$\chi^2(\rho_0, \Sigma_0) = \sum_{i=1}^{N} \left(\left[v_i^{\text{data}}\right]^2 - \left[v_i^{\text{theory}}(\rho_0, \Sigma_0)\right]^2 \right)^2 ,$$

where N is the number of data points, v_i^{data} is the i^{th} measurement of the velocity, v_i^{theory} is the predicted velocity at the i^{th} radius in the table. It is a function of ρ_0, Σ_0. There are two things a little different about this χ^2 from normal ones. First, usually there's an error squared in the denominator. They didn't give error bars, so we'll assume they're the same at every point. Second, I'm asking you to compute the difference between the theoretical and observational velocities *squared.* This allow you to make some progress analytically. Find the values of (Σ_0, ρ_0) which minimize this χ^2. These are the best fit values (theoretical predictions closest to the data).

4. Problem 23.11 in Carroll-Ostlie.